

READER

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Chances Are, It's Friday the 13th

A Case Study in Mathematical Obsession

By Lawrence Weschler

"The thirteenth of the month is more likely to occur on a Friday than on any other single day of the week," or anyway that's what this bespectacled stranger on the bus insisted to me about ten days ago—"and you can prove it," is what he said. I don't suppose he meant this last remark as a challenge, and I doubt he gave the conversation a second thought (he no sooner dropped the gauntlet than he disembarked the bus), but there it lay, pulsing, all the way down Wilshire Boulevard, and when I got off the bus I ended up picking it up and taking it home with me.

For the next week it took over my life. Listen, I'm no mathematician. I'm just a struggling free-lance writer, and I don't have time for this sort of nonsense—I've got early deadlines to meet and bare cupboards to fill. Listen, I said to myself all week long, as the files of figures invaded my dreams and the sheaves of notes overflowed my desk (by the end of the third day I couldn't even find my drafts for the article I was supposed to be writing—I think I threw them out on the backside of a particularly unproductive line of reasoning), listen, I said—you don't have time for this.

But a puzzle can get to be like a fever, and it will run its course. Anyway, I think I've got it now, finally, the proof, and I've even managed to sanitize it of much of the hysteria that accompanied its formulation. So what follows is in the form of a message to that stranger on the bus: I did it, OK? Satisfied? I'm sure there must be easier ways to do it, and I'm sure you can think of half a dozen of them, but you know what I think? I wish that you and *your* proof would just both drop dead.



CALENDARS — 1776 to 2000

DIRECTIONS FOR USE

Look for the year you want in the index at left. The number opposite each year is the number of the calendar to use for that year.

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1797	1	1822	3	1847	6	1872	9	1897	6	1922	1	1947	4	1972	14	1997	4
1798	2	1823	4	1848	14	1873	4	1898	7	1923	2	1948	12	1973	2	1998	5
1799	3	1824	12	1849	2	1874	5	1899	1	1924	10	1949	7	1974	3	1999	6
1800	4	1825	7	1850	3	1875	6	1900	2	1925	5	1950	1	1975	4	2000	14

Friday the 13th

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We can prove this last contention in the following manner: Three phases of 40 years add up to 120 years; 400 minus 120 yields 280; 280 divided by 28 yields 10. Thus in any period of 400 years, we will have an **A**, a **C**, and a **D** sequence, each of 40 years, interspersed with 10 **B** sequences of 28 years. They will occur in the following order:

BBDBBABCBBBB

On the chart 1776-2000 that we are employing, we cut in beginning with an **A** pattern in 1776:

1800 1900 2000

1776 **A** 1816 **B** 1844 **B** 1872 **C** 1912 **B** 1940 **B** 1968 **B** 1996 **B** 2024

We can, by simple subtraction, extend our chart back to 1624:

1700

1624 **B** 1652 **B** 1680 **D** 1720 **B** 1748 **B** 1776 **A**

We note, of course, that the first date, 1624, is 400 years earlier than the last date, 2024, and that taking the sequence further back we would only begin to repeat the pattern. Four hundred years is the phase.

And, of course, this stands to reason, since by its very rules (see I.b.2) our calendar goes in phases of 400 years.

IV. a) Therefore, we have isolated four patterns, **A B C** and **D**, that comprise all the possible sequences in our calendar. These sequences are:

B 9,4,5,6,14,2,3, 4,12,7,1,2,10,5,6,7,8,3,4,5,13,1,2,3,11,6,7,1.

A 9.4.5.6.14.2.3.4.12.7.1.2.10.5.6.7.8.3.4.5.13.1.2.3// 4,5,6,7,8,

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teen calendars with its assigned color. This will keep us busy for a

occur on a Friday than on any other of the days of the week.

A 9,4,5,6,14,2,3,4,12,7,1,2,10,5,6,7,8,3,4,5,13,1,2,3,11,6,7,1,
 3,4,5,13,1,2,3,11,6,7,1.
 C 9,4,5,6,14,2,3,4,12,7,1,2,10,5,6,7,8,3,4,5,13,1,2,3,11,6,7,1//
 2,3,4,5,13,1,2,3,11,6,7,1.
 (Note: // indicates centesimal.)

We have solved these with relative ease, because we used the examples in our chart: **B** 1816-1843; **A** 1776-1815; and **C** 1872-1911. But our chart does not provide us with an example of **D** (1680-1719), so we will have to manufacture our own.

b) How to manufacture a chart for the years 1680-1719 (**D**). Begin in 1680 with the opening phrase ("9,4,5,6...") and continue using pattern **B** up to 1699. Then, locate which number calendar applies to 1699, consult that calendar, see which day it ends on, find the number of the calendar of the standard year that begins the following day, and that's the number for 1700. Repeat the process for each consecutive year, making sure that on leap years you use leap calendars. The result:

1680	9	1690	1	1700	6	1710	4
1681	4	1691	2	1701	7	1711	5
1682	5	1692	10	1702	1	1712	13
1683	6	1693	5	1703	2	1713	1
1684	14	1694	6	1704	10	1714	2
1685	2	1695	7	1705	5	1715	3
1686	3	1696	8	1706	6	1716	11
1687	4	1697	3	1707	7	1717	6
1688	12	1698	4	1708	8	1718	7
1689	7	1699	5	1709	3	1719	1

This sequence **D** will run as follows:

D 9,4,5,6,14,2,3,4,12,7,1,2,10,5,6,7,8,3,4,5,13,1,2,3,11,6,7,1,
 3,4,5,13,1,2,3,11,6,7,1.

c) We will omit here a discussion of all of the interesting patterns within each of these four sequences (those that result from the parallel 7 through 8 and 14 sets of possible calendars, and the fact that the first in January in consecutive standard years pops forward one day of the week). They are readily apparent, but unimportant in terms of our concerns.

We have, however, in summary, established the sequence within four possible patterns **A B C D** and shown the repeating cycle of these patterns to run through 400-year phases. We have also shown that the given **A B C D** sequences are the only possible sequences since the 400-year phase is a closed one.

Now let us turn to the vexing problem of the 13th day of the month.

V. We ask ourselves, given the fourteen possible variables of standard and leap-year calendars, on what days of the week does the 13th of each month tend to fall? More precisely, within each of the fourteen possible year-calendars, how many times does each different day of the week (Monday, Tuesday, Wednesday, etc.) fall on a 13th?

This question can be answered through a simple, if somewhat extended and laborious, process. Let us go through the chart of the fourteen possible calendars with pens of seven different colors (one for each day, Sunday is red, Monday is blue—ain't it the truth? Tuesday is green, etc.), daubing every 13th of the month in all four-

teen calendars with its assigned color. This will keep us busy for a while, but when we're finished, let us compile a chart whose horizontal will be the calendars 1 through 14, and whose vertical will be "the days of the week upon which the 13th falls" (i.e., Sunday through Saturday). We shall call this chart:

SCHEME 1

A chart of how many times the 13th of the month falls on each day of the week within each of the fourteen different types of calendars.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
SUNDAY	1	1	2	2	2	1	3	1	1	3	2	1	2	2
MONDAY	3	1	1	2	2	2	1	1	3	2	1	2	1	2
TUESDAY	1	3	1	1	2	2	2	2	1	1	3	2	1	2
WEDNESDAY	2	1	3	1	1	2	2	1	2	1	1	3	2	2
THURSDAY	2	2	1	3	1	2	2	2	1	2	2	1	1	3
FRIDAY	2	2	2	1	3	1	3	2	1	2	2	1	1	1
SATURDAY	1	2	2	1	3	1	1	3	2	1	2	2	1	2

Note: This scheme may be checked in the following ways:

a) Each number calendar has twelve 13ths of the month (one per month), so that each vertical column should, and does, add up to 12. Indeed, each vertical column includes one 3s, three 2s, and three 1s.

b) Because calendars 1 through 14 include every possible configuration, the total number of times each individual day of the week appears on the 13th will be equal across the expanse of all fourteen calendars. This number will be 24, or 12 standard times plus 12 leap times. Every horizontal sequence of numbers therefore should, and does, add up to 24. Indeed, every horizontal file includes two 3s, six 2s, and six 1s.

VI. Now let us return to a consideration of our four patterns **A B C** and **D**. We should now ask how many times each of the fourteen constituent integers appears in each sequence of forty or twenty-eight numbers. This is simply a matter of counting through the four patterns as they are recorded in section IV. Our result is:

SCHEME 2

The number of times each of the fourteen types of calendars appears in each of the four patterns.

	1	2	3	4	5	6	7	8	9	10	11	12	13	14
B	3	3	3	3	3	3	3	1	1	1	1	1	1	1
A	4	4	5	5	4	4	2	1	1	1	1	2	1	1
C	5	5	4	4	4	4	1	1	1	2	1	2	1	1
D	4	4	4	4	5	5	2	1	2	1	1	1	1	1

Note: The check here, of course, is that each horizontal column should, and does, add up to the appropriate number of integers (that is, 28 for **B** and 40 for **A C** and **D**).

VII. At last we approach our solution. Our question is this: In the 400-year period that constitutes our cycle, how many times does the 13th fall on each day of the week? If we can show that it falls more often on Friday than on any other individual day, then we will have proved our contention that the 13th of the month is more likely to

occur on a Friday than on any other of the days of the week.

We can do this in the following way:

We recall that 400 years disperse themselves through the following formula:

$$10B + A + C + D = 400 \text{ years.}$$

Let us first determine the distribution of 13ths of the month within **B**, **A**, **C**, and **D** respectively, and then run the results through our formula.

This can be accomplished through an intersection of our two schemes. Scheme 1 tells us how many times any given day (for example, Friday) occurs on the 13th during each of the fourteen possible calendars. Scheme 2 tells us how often the fourteen possible calendars occur respectively in our four possible patterns **A B C** and **D**. To determine how many times the 13th falls on Day X in Pattern A, therefore, align the A values for 1 through 14 in Scheme 2 above the X values for 1 through 14 in Scheme 1, and then multiply each of the fourteen (vertical) pairs of numbers and add up the fourteen resultant totals (horizontally). For example, let us take the example of Friday A:

Calendars:	1	2	3	4	5	6	7	8	9	10	11	12	13	14
A values:	4	4	5	5	5	4	4	2	1	1	1	1	2	2
Friday values:	2	2	2	1	3	1	1	3	2	1	2	2	1	1
Totals:	8	8	10	5	15	4	4	6	2	1	2	2	2	1
Grand Total =	70													

(What this chart shows us, for example, in its first vertical column, is that the 1 calendar appears four times in Sequence A, and that the 13th of the month is a Friday twice in that 1 calendar, and that therefore there are eight Friday-the-13ths in 1-type calendars in the A sequence.) The grand total will be the number of times that any given day of the week appears as the 13th of the month during that particular sequence. In this particular example, we have determined that there are seventy Friday-the-13ths in Sequence A (there would thus be seventy Friday-the-13ths between 1776 and 1815, for example).

Now, by repeating the process through multiple small additions and multiplications, we can arrive at the figures in:

SCHEME 3

The number of times the 13th of the month occurs on each day of the week in each of the four possible sequences.

	B	A	C	D
SUNDAY	48	69	68	70
MONDAY	48	68	69	68
TUESDAY	48	68	68	69
WEDNESDAY	48	69	70	68
THURSDAY	48	68	68	68
FRIDAY	48	70	69	69
SATURDAY	48	68	68	68

Note: To check Scheme 3, consider that **B** consists of twenty-eight years, while **A**, **C**, and **D** consist of forty years each. The totals of the vertical columns should equal twelve times the number of years

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in each pattern, since each year has twelve months and each month has one 13th. They do. (The **B** column consists of $7 \times 48 = 336$, and 336 in turn equals 28×12 . The **A**, **C**, and **D** columns all consist of $70 + 69 + 69 + 68 + 68 + 68 + 68 = 480$, which in turn equals 40×12 .)

Now, recalling the formula

$$10\mathbf{B} + \mathbf{A} + \mathbf{C} + \mathbf{D} = 400$$

we can produce our final scheme, Scheme 4, by multiplying the **B** figures in Scheme 3 by ten and then adding the figures in the three other columns horizontally. Our result will be how many times in 400 years the 13th of the month occurs on each given day of the week.

SCHEME 4

The number of times the 13th of the month occurs on each day of the week over a phase of 400 years.

	10 B	A	C	D	= Grand Total
SUNDAY	480	69	68	70	687
MONDAY	480	68	69	68	685
TUESDAY	480	68	68	69	685
WEDNESDAY	480	69	70	68	687
THURSDAY	480	68	68	68	684
FRIDAY	480	70	69	69	688 . . . da winnah!
SATURDAY	480	68	68	68	684



Thus, over a period of 400 years (which we have determined to be the period of our repeating cycle), the 13th of the month will fall on a Friday more times than on any other single day (once more, that is, than on either Wednesday or Sunday).

Hence, it is true: The 13th of the month is more likely to occur on a Friday than on any other single day of the week.

JUST BARELY.

(And of course, this is a complete fluke, owing entirely to the fact that today, July 13, 1979 is a Friday, rather than, say, a Wednesday, in which case the day with the one-point margin of 688 would have been some other day altogether.

Quiz: Which one?)